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Spiky strings on $AdS_4 \times CP^3$

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Abstract: We study a giant magnon and a spike solution for the string rotating on $AdS_4 \times \mathbf{CP}^3$ geometry. We consider rigid rotating fundamental string in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ sector inside the $\mathbf{CP}^3$ and find out the general form of all the conserved charges. We find out the dispersion relation corresponding to both the known giant magnon and the new spike solutions. We further study the finite size correction in both cases.

Keywords: Long strings, Gauge-gravity correspondence.

Contents

1. Introduction and summary

The AdS/CFT duality [1] relates type IIB string theory on $AdS_5 \times S^5$ with $\mathcal{N} = 4$ superconformal Yang-Mills (SYM) theory, and it has been celebrated in the last decade as one of the exact duality between string and gauge theory. Recently there has been a lot of works devoted towards the understanding of the worldvolume dynamics of multiple M2-branes, initiated by Bagger, Lambert and Gustavsson [2] based on the structure of Lie 3-algebra. In this new development of understanding of the worldvolume theory of coincident M-branes in M-theory, a new class of conformal invariant 2+1 dimensional field theories has been found out. Based on this Aharony, Bergman, Jafferis and Maldacena (ABJM) [3] proposed a new gauge-string duality between $\mathcal{N} = 6$ Chern-Simons theory and type IIA string theory on $AdS_4 \times \mathbf{CP}^3$. More precisely, ABJM theory has been conjectured to be dual to M-theory on $AdS_4 \times S^7/Z_k$ with N units of four-form flux which for $k \ll N \ll k^5$ can be compactified to type IIA theory on $AdS_4 \times \mathbf{CP}^3$, where k is the level of Chern-Simon theory with gauge group SU(N). This ABJM theory is weakly coupled for $\lambda \ll 1$, where $\lambda = N/k$ is the 't Hooft coupling. Once this duality was proposed, there has been a numerous effort in understanding the ABJM theory more [4]–[28].

In the development of AdS_5/CFT_4 duality, an interesting observation is that the $\mathcal{N} = 4$ SYM theory in planar limit can be described by an integrable spin chain model where the anomalous dimension of the gauge invariant operators were found [29 – 33]. It was further noticed that the string theory is also integrable in the semiclassical limit [34 – 38] and the anomalous dimension of the $\mathcal{N} = 4$ SYM can be derived from the relation between conserved charges of the rotating string $AdS_5 \times S^5$. In this connection, Hofman and Maldacena (HM) [39] considered a special limit where the problem of determining the spectrum in both sides becomes rather simple. The spectrum consists of an elementary

excitation known as magnon that propagates with a conserved momentum p along the infinitely long [40 – 45, 35, 46 – 51] or the finitely long [22, 52 – 54] spin chain. In the dual formulation, the most important ingredient is the semiclassical string solution, which can be mapped to long trace operator with large energy and large angular momenta. A more general class of rotating string solutions are the spiky strings [55, 56] that describe the higher twist operators from the field theory view point. Giant magnon solutions could be thought of as a special limit of such spiky strings with shorter wavelength. Recently there has been a lot of work devoted for finding the giant magnon and spike solutions for strings in more general background, (see for example [57 – 61]).

The integrability of AdS_5/CFT_4 in the planar limit using a Bethe ansatz brings us the hope that the recently proposed AdS_4/CFT_3 duality will also be solvable by using a similar ansatz [9]. Indeed, in [9, 10, 12, 13] this has been investigated and many interesting results were found. The magnon solutions were found in the SU(2) × SU(2) sub-sector of $\mathbf{CP}^3$. For example the giant magnon found in [15, 21] is a solitonic string living on $R \times S^2 \times S^2$ and rotating uniformly around the two spheres. In the present paper we would like to find out a spike solution for the string rotating on $S^2 \times S^2$, and interpret it as a general class of solution in the worldsheet theory. We solve the equations of motion and the Virasoro constraints for the Polyakov action of the string. We write down the general form of equations of motion which in two different limits corresponds to the already known giant magnon and the new spike solution for the string moving in the SU(2) × SU(2). The dispersion relations among the various conserved charges have been found out in both cases. We further study the finite size corrections to the dispersion relations.

The rest of the paper is organized as follows. In section 2. we consider a rotating string solution on $R \times S^2 \times S^2$, which is obtained by fixing some coordinates of $AdS_4 \times \mathbf{CP}^3$. Taking into account the Polyakov form of the action for the string in this background, we find the general forms of all conserved charges. In section 3, we find out the giant magnon and spike as two different limiting cases and write the dispersion relation along various conserved charges. For the magnon case, we reproduce the result obtained in [15, 21]. Section 4 is devoted to the study of finite size effects for both the giant magnon and spike solutions. In section 5, we present our conclusions.

2. Rotating strings on $R \times S^2 \times S^2$

In this section, we will investigate a general class of rotating string solution on $R \times S^2 \times S^2$ which is a subspace of $AdS_4 \times \mathbf{CP}^3$. We start by writing down the metric for $AdS_4 \times \mathbf{CP}^3$

$$\begin{aligned} ds^2 = & \frac{1}{4}R^2\left[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \\ & + R^2\left[d\xi^2 + \cos^2\xi\sin^2\xi\left(d\psi + \frac{1}{2}\cos\theta_1 d\phi_1 - \frac{1}{2}\cos\theta_2 d\phi_2\right)^2\right. \\ & \left. + \frac{1}{4}\cos^2\xi\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \frac{1}{4}\sin^2\xi\left(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right)\right]. \quad (2.1) \end{aligned}$$

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While taking $\alpha' = 1$, the curvature radius R is given by $R^2 = 2^{5/2}\pi\lambda^{1/2}$. The 't Hooft coupling constant is $\lambda \equiv N/k$ where k is the level of the 3-dimensional $\mathcal{N} = 6$ ABJM model.

To investigate the string theory dual to spin chain model of SU(2) sector in the boundary SYM, we first consider the string moving in $R \times S^2 \times S^2$, which is the subspace of $R \times \mathbf{CP}^3$ and corresponds to the SU(2)×SU(2) R-symmetry group of the boundary SYM. This subspace can be obtained by choosing $\rho = 0$, ψ and ξ =constant and then giving the identification $\theta_1 = \theta_2 \equiv \theta$ and $\phi_1 = \phi_2 \equiv \phi$. Note that this identification reduces $R \times S^2 \times S^2$ to $R \times S'^2$ effectively, where S'^2 can be parameterized by $\theta = \frac{1}{2}(\theta_1 + \theta_2)$ and $\phi = \frac{1}{2}(\phi_1 + \phi_2)$ and corresponds to the diagonal SU(2) subgroup of SU(2) × SU(2) R-symmetry.

More precisely, the action for string moving in $R \times \mathbf{CP}^3$, where R is the time direction on AdS_4 at $\rho = 0$, is

$$\begin{aligned} S = \frac{R^2}{16\pi}\int d^2\sigma \big[& -\partial_\alpha t\partial^\alpha t + 4\partial_\alpha\xi\partial^\alpha\xi + 4\cos^2\xi\sin^2\xi\Gamma_\alpha\Gamma^\alpha \\ & + \cos^2\xi(\partial_\alpha\theta_1\partial^\alpha\theta_1 + \sin^2\theta_1\partial_\alpha\phi_1\partial^\alpha\phi_1) \\ & + \sin^2\xi(\partial_\alpha\theta_2\partial^\alpha\theta_2 + \sin^2\theta_2\partial_\alpha\phi_2\partial^\alpha\phi_2)\big] \end{aligned} \tag{2.2}$$

with

$$\Gamma_\alpha = \partial_\alpha\psi + \frac{1}{2}\cos\theta_1\partial_\alpha\phi_1 - \frac{1}{2}\cos\theta_2\partial_\alpha\phi_2, \tag{2.3}$$

where α, β implies the string worldsheet indices. The equations of motion for ξ and ψ are

$$\begin{aligned} 0 = {} & 4\partial^\alpha\partial_\alpha\xi - 4\sin 2\xi\cos 2\xi\Gamma_\alpha\Gamma^\alpha + \sin\xi\cos\xi(\partial_\alpha\theta_1\partial^\alpha\theta_1 \\ & + \sin^2\theta_1\partial_\alpha\phi_1\partial^\alpha\phi_1 - \partial_\alpha\theta_2\partial^\alpha\theta_2 - \sin^2\theta_2\partial_\alpha\phi_2\partial^\alpha\phi_2), \\ 0 = {} & \partial^\alpha(\cos^2\xi\sin^2\xi\Gamma_\alpha). \end{aligned} \tag{2.4}$$

When ψ = constant, $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$ gives $\Gamma_\alpha = 0$, which satisfies the second equation in eq. (2.4) and reduces the first equation to a simple form $0 = 4\partial^\alpha\partial_\alpha\xi$. The simplest solution of this is ξ = constant. Under these solutions, the open string motion on $\mathbf{CP}^3$ reduces to that on S'^2 effectively. Therefore, the rest equations of motion for other fields become

$$\begin{aligned} 0 = {} & \partial^\alpha\partial_\alpha t, \\ 0 = {} & \partial^\alpha\partial_\alpha\theta - 2\sin\theta\cos\theta\partial_\alpha\phi\partial^\alpha\phi, \\ 0 = {} & \partial^\alpha\left(\sin^2\theta\partial_\alpha\phi\right). \end{aligned} \tag{2.5}$$

Note that these equations are those for the string sigma model moving on $R \times S^2 \times S^2$ with constraints $\theta_1 = \theta_2 = \theta$ and $\phi_1 = \phi_2 = \phi$.

The reduction from $\mathbf{CP}^3$ to $S^2 \times S^2$ can be shown with different way using the complex coordinates. For that, we first consider the embedding S^7 to R^8. Then, S^7 can be described by the constraint equation, in terms of complex coordinates Z_i $(i = 1, \cdots, 4)$ or real coordinates X_a $(a = 1, \cdots, 8)$ on the 8-dimensional Euclidean space,

$$\frac{R^2}{4} = \sum_{i=1}^{4}|Z_i|^2 = \sum_{a=1}^{8}X_a^2, \tag{2.6}$$

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where we set $Z_i = X_i + iX_{i+4}$. Imposing more constraint

$$0 = \frac{i}{2}\sum_{i=1}^{4}(Z_i\partial_\alpha\bar{Z}_i - \bar{Z}_i\partial_\alpha Z_i) = \sum_{i=1}^{4}(X_i\partial_\alpha X_{i+4} - X_{i+4}\partial_\alpha X_i), \tag{2.7}$$

where α implies the world sheet indices, reduces the above S^7 to $\mathbf{CP}^3$ [21]. The complex coordinates representing $S^2 \times S^2$ become in terms of the angular variables in eq. (2.5),

$$\begin{aligned} Z_1 &= \frac{R}{2}\cos\xi\sin\theta e^{i\phi}, \\ Z_2 &= \frac{R}{2}\cos\xi\cos\theta, \\ Z_3 &= \frac{R}{2}\sin\xi\sin\theta e^{-i\phi}, \\ Z_4 &= \frac{R}{2}\sin\xi\cos\theta, \end{aligned} \tag{2.8}$$

where ξ is a constant. This parameterization satisfies the constraint for S^7 and to satisfy the constraint for $\mathbf{CP}^3$ in eq. (2.7) we should set $\xi = \frac{\pi}{4}$. This effectively describes S'^2 as the subspace of $\mathbf{CP}^3$.

The Polyakov action for a string moving on this $R \times S^2 \times S^2$ is given by

$$S = \frac{1}{4\pi}\int d^2\sigma\ \sqrt{-\det h}\ h^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}, \tag{2.9}$$

with the metric

$$ds^2 = \frac{1}{4}R^2\left[-dt^2 + \cos^2\xi\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \sin^2\xi\left(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right)\right], \tag{2.10}$$

which reduces to $R \times S'^2$ under the identification, $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$.

In terms of target space coordinates in the conformal gauge $h^{\alpha\beta} = \eta^{\alpha\beta}$, the effective action on $R \times S'^2$ is written as

$$S = \frac{T}{2}\int d^2\sigma\left[(\partial_\tau t)^2 - (\partial_\sigma t)^2 - (\partial_\tau\theta)^2 + (\partial_\sigma\theta)^2 - \sin^2\theta\left\{(\partial_\tau\phi)^2 - (\partial_\sigma\phi)^2\right\}\right], \tag{2.11}$$

where the string tension T is given by

$$T = \frac{\sqrt{2\lambda}}{2}\ . \tag{2.12}$$

To find the giant magnon or spike solutions for string, we choose the following parametrization

$$\begin{aligned} t &= f(\tau), \\ \theta_1 = \theta_2 &= \theta(y), \\ \phi_1 = \phi_2 = \phi &= \nu\tau + g(y), \end{aligned} \tag{2.13}$$

where $y = a\tau + b\sigma$ and τ and σ run from $-\infty$ to ∞.

Due to the translation symmetry along t and the rotational symmetry along ϕ_i's ($i = 1, 2$), there exist three conserved charges and the equations of motion for the corresponding fields are given by

$$\begin{aligned} 0 &= \partial_\tau^2 f(\tau), \\ 0 &= \partial_y \left[\sin^2\theta \left\{a\nu + (a^2 - b^2) g'\right\}\right], \end{aligned} \tag{2.14}$$

where prime implies derivative with respect to y. The solutions of these equations are

$$\begin{aligned} f(\tau) &= \kappa\tau\,, \\ g'(y) &= \frac{1}{a^2 - b^2} \frac{c - a\nu\sin^2\theta}{\sin^2\theta}\,, \end{aligned} \tag{2.15}$$

where κ and c are integration constants. The equation of motion for the world sheet metric $h^{\alpha\beta}$, gives rise to the Virasoro constraints $T_{\alpha\beta} = 0$ where

$$\begin{aligned} T_{\alpha\beta} &\equiv \frac{1}{\sqrt{-\det h}} \frac{\partial \mathcal{L}}{\partial h^{\alpha\beta}} \\ &= \frac{T}{2}\left(\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu} - \frac{1}{2}\eta_{\alpha\beta}\eta^{\alpha'\beta'}\partial_{\alpha'} x^\mu \partial_{\beta'} x^\nu G_{\mu\nu}\right). \end{aligned} \tag{2.16}$$

Due to the symmetric property of the metric, the independent constraints are three, $T_{\tau\tau}$, $T_{\tau\sigma}$ and $T_{\sigma\sigma}$. Furthermore, the conformal nature of the Polyakov action gives rise to the relation $T_{\tau\tau} = T_{\sigma\sigma}$, so only two of them are independent. For later convenience, these two Virasoro constraints are rewritten as

$$\begin{aligned} 0 &= T_{\tau\tau} + T_{\sigma\sigma} + 2T_{\tau\sigma}\,, \\ 0 &= T_{\tau\tau} + T_{\sigma\sigma} - \frac{a^2 + b^2}{ab} T_{\tau\sigma}. \end{aligned} \tag{2.17}$$

The first line of eq. (2.17) gives a first order differential equation for θ,

$$\theta' = \frac{b\nu}{a^2 - b^2} \frac{\sqrt{(\sin^2\theta_{\rm max} - \sin^2\theta)(\sin^2\theta - \sin^2\theta_{\rm min})}}{\sin\theta}, \tag{2.18}$$

where $\sin\theta_{\rm max}$ and $\sin\theta_{\rm min}$ satisfy

$$\begin{aligned} \sin^2\theta_{\rm max} + \sin^2\theta_{\rm min} &= \frac{\kappa^2(a-b)^2 + 2b\nu c}{b^2\nu^2}, \\ \sin^2\theta_{\rm max} \cdot \sin^2\theta_{\rm min} &= \frac{c^2}{b^2\nu^2}. \end{aligned} \tag{2.19}$$

The second line of eq. (2.17) reduced to a relation among various constants. From this, one can find

$$a = \frac{\nu}{\kappa^2} c. \tag{2.20}$$

Using this, we finally obtain

$$\theta' = \frac{b\nu}{a^2 - b^2} \frac{\sqrt{(\frac{c^2}{\kappa^2 b^2} - \sin^2\theta)(\sin^2\theta - \frac{\kappa^2}{\nu^2})}}{\sin\theta}. \tag{2.21}$$

Now let us proceed to write various conserved charges for this system. The energy is given by

$$\begin{aligned} E &\equiv T\int d\sigma\partial_\tau t \\ &= 2T\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\theta\ \frac{\kappa(a^2-b^2)}{b^2\nu}\frac{\sin\theta}{\sqrt{(\sin^2\theta_{\rm max}-\sin^2\theta)(\sin^2\theta-\sin^2\theta_{\rm min})}} \end{aligned} \tag{2.22}$$

and two angular momenta are

$$\begin{aligned} J_1 &\equiv -T\cos^2\xi\int d\sigma\sin^2\theta\partial_\tau\phi \\ &= -2T\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\theta\frac{1}{2b^2\nu}\frac{\sin\theta(ac-b^2\nu\sin^2\theta)}{\sqrt{(\sin^2\theta_{\rm max}-\sin^2\theta)(\sin^2\theta-\sin^2\theta_{\rm min})}}, \\ J_2 &\equiv -T\sin^2\xi\int d\sigma\sin^2\theta\partial_\tau\phi \\ &= -2T\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\theta\frac{1}{2b^2\nu}\frac{\sin\theta(ac-b^2\nu\sin^2\theta)}{\sqrt{(\sin^2\theta_{\rm max}-\sin^2\theta)(\sin^2\theta-\sin^2\theta_{\rm min})}}. \end{aligned} \tag{2.23}$$

Note that J_1 and J_2 are angular momentum on each sphere S^2. To consider a giant magnon or spike solution, we have to define the world sheet momentum p, which is identified with the angle difference $\Delta\phi \equiv p$,

$$\begin{aligned} \Delta\phi &\equiv -\int d\phi = -2\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\theta\frac{g'}{\theta'} \\ &= -2\int_{\theta_{\rm min}}^{\theta_{\rm max}} d\theta\frac{1}{b^2\nu}\frac{(bc-ab\nu\sin^2\theta)}{\sin\theta\sqrt{(\sin^2\theta_{\rm max}-\sin^2\theta)(\sin^2\theta-\sin^2\theta_{\rm min})}}, \end{aligned} \tag{2.24}$$

where we use a minus sign for making the angle difference positive.

3. Giant magnon and spike solutions

By using various quantities defined in the previous section, we now proceed to find the relation among various conserved charges. Before doing this, first we choose the infinite size limit, which implies infinite angular momentum in case of giant magnon and infinite angle between two end points of a spike. Note that this infinite size limit can be described by setting $\sin\theta_{\rm max} = 1$ in both cases. In this case, eq. (2.18) is reduced to

$$\theta' = \frac{b\nu}{a^2-b^2}\frac{\cos\theta\sqrt{(\sin^2\theta-\sin^2\theta_{\rm min})}}{\sin\theta}. \tag{3.1}$$

Due to the cosine term in the above equation, all the conserved charges diverge except for a special region of parameters. Below, we will investigate the solutions in this region.

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3.1 Giant magnon solution

In this section, we will consider a magnon solution which has infinite charges, E and $J \equiv J_1 + J_2$, but finite difference of them, $E - J$. For this, we should choose from eq. (2.21)

$$\sin\theta_{\max} = -\frac{\kappa}{\nu}, \tag{3.2}$$

where the minus sign is inserted for later consistency. In the infinite size limit ν can be rewritten as

$$\nu = -\kappa, \tag{3.3}$$

where we assume that κ is positive. Using eq. (2.20) and eq. (3.3), $E - J$ is given by

$$E - J = 2T \int_{\theta_{\min}}^{\pi/2} d\theta \frac{\sin\theta\cos\theta}{\sqrt{(\sin^2\theta - \sin^2\theta_{\min})}} = 2T\sqrt{1 - \sin^2\theta_{\min}}, \tag{3.4}$$

where $\sin\theta_{\min} = c/\kappa b$. Note that $E - J$ has no divergence like our expectation. The value of the world sheet momentum p corresponding to the angle difference is also obtained using eq. (2.20) and eq. (3.3)

$$\Delta\phi = 2\int_{\theta_{\min}}^{\pi/2} d\theta \frac{c}{\kappa b} \frac{\cos\theta}{\sin\theta\sqrt{(\sin^2\theta - \sin^2\theta_{\min})}} = 2\arccos(\sin\theta_{\min}). \tag{3.5}$$

Finally, we obtain the dispersion relation for a giant magnon as

$$E - J = \sqrt{2\lambda}\left|\sin\frac{p}{2}\right|, \tag{3.6}$$

where we replace the string tension T with the 't Hooft coupling λ. This is the dispersion relation for an open string rotating in S'^2 effectively, which is dual to the open spin chain in the SU(2) diagonal R-symmetry subgroup.

In ref. [12, 15], a single trace operator corresponding to a closed spin chain in SU(2) × SU(2) sector is considered. Moreover, it was shown that the dual string solution for this closed spin chain is a closed string rotating in $S^2 \times S^2$, which is a combination of two open strings, each rotating on different S^2. In ref. [12, 15, 21], the open string corresponding to half of the closed string has also the same dispersion relation in eq. (3.6) but the angular momentum J is given by $J = J_1$ or J_2. However, in this case the giant magnon describes not the open spin chain in the diagonal SU(2) but that in one of the SU(2) inside SU(2)×SU(2) R-symmetry group.

3.2 Spike solution

To find a spike solution, we should impose that J is finite. For this, we choose $\sin\theta_{\max} = \frac{c}{\kappa b}$ in eq. (2.21). In the infinite size limit, κ can be rewritten as in terms of c and b

$$\kappa = \frac{c}{b}. \tag{3.7}$$

Then, $\sin\theta_{\min}$ becomes

$$\sin\theta_{\min} = \frac{c}{b\nu} \tag{3.8}$$

Using these, $E - T\Delta\phi$ and J become

$$E - T\Delta\phi = 2T \int_{\theta_{\rm min}}^{\pi/2} d\theta \frac{\sin\theta_{\rm min}\cos\theta}{\sin\theta\sqrt{(\sin^2\theta - \sin^2\theta_{\rm min})}} = 2T\arccos(\sin\theta_{\rm min}),$$
$$J = -2T \int_{\theta_{\rm min}}^{\pi/2} d\theta \frac{\sin\theta\cos\theta}{\sqrt{(\sin^2\theta - \sin^2\theta_{\rm min})}} = -2T\sqrt{1-\sin^2\theta_{\rm min}}. \tag{3.9}$$

Notice that if the orientation of the rotation in ϕ_i-direction ($i = 1, 2$), is changed, then we can obtain a positive J. From now on, we consider a positive J. Then, $E - T\Delta\phi$ and J can be rewritten in terms of new variable $\tilde{\theta} = \pi/2 - \theta_{\rm min}$ as

$$E - T\Delta\phi = \sqrt{2\lambda}\tilde{\theta},$$
$$J = \sqrt{2\lambda}\sin\tilde{\theta}, \tag{3.10}$$

with $J = J_1 + J_2$

$$J_1 = \frac{\sqrt{2\lambda}}{2}\sin\tilde{\theta},$$
$$J_2 = \frac{\sqrt{2\lambda}}{2}\sin\tilde{\theta}. \tag{3.11}$$

4. Finite size effects

In the previous section, we found a giant magnon and a spike solution in the infinite size limit. Here, we will investigate the finite size effect on them.[1] To do so, we have to investigate the solitonic string solution when $\theta_{\rm max} \neq \pi/2$.

Giant magnon case: for the magnon case, $\sin\theta_{\rm min}$ and $\sin\theta_{\rm max}$ become

$$\sin\theta_{\rm max} = -\frac{\kappa}{\nu},$$
$$\sin\theta_{\rm min} = \frac{c}{\kappa b}, \tag{4.1}$$

For the simple calculation, we replace the variable θ to $z \equiv \cos\theta$. With this new variable z, the conserved charges are rewritten as

$$E = 2T\ \frac{z_{\rm max}^2 - z_{\rm min}^2}{z_{\rm max}\sqrt{1-z_{\rm min}^2}}\ K(x),$$
$$J = 2Tz_{\rm max}\ \left[K(x) - E(x)\right],$$
$$\frac{\Delta\phi}{2} = \frac{\sqrt{1-z_{\rm min}^2}}{z_{\rm max}\sqrt{1-z_{\rm max}^2}}\Pi\left(\frac{z_{\rm max}^2 - z_{\rm min}^2}{\sqrt{z_{\rm max}^2-1}}; x\right) - \frac{\sqrt{1-z_{\rm max}^2}}{z_{\rm max}\sqrt{1-z_{\rm min}^2}}K(x), \tag{4.2}$$

[1] Finite size effect for the membrane in $AdS_4\times S^7$ has been investigated in ref. [62].

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by using the elliptic integrals of the first, second and third kinds

$$
\begin{aligned}
K(x) &= \int_{z_{\rm min}}^{z_{\rm max}} dz \frac{z_{\rm max}}{\sqrt{(z_{\rm max}^2 - z^2)(z^2 - z_{\rm min}^2)}}, \\
E(x) &= \int_{z_{\rm min}}^{z_{\rm max}} dz \frac{z^2}{z_{\rm max}\sqrt{(z_{\rm max}^2 - z^2)(z^2 - z_{\rm min}^2)}}, \\
\Pi\left(\frac{z_{\rm max}^2 - z_{\rm min}^2}{\sqrt{z_{\rm max}^2 - 1}}; x\right) &= \int_{z_{\rm min}}^{z_{\rm max}} dz \frac{z_{\rm max}(1 - z_{\rm max}^2)}{(1 - z^2)\sqrt{(z_{\rm max}^2 - z^2)(z^2 - z_{\rm min}^2)}},
\end{aligned} \tag{4.3}
$$

where $z_{\rm max}^2 \equiv \cos^2\theta_{\rm min} = \frac{k^2b^2 - c^2}{\kappa^2 b^2}$, $z_{\rm min}^2 \equiv \cos^2\theta_{\rm max} = \frac{\nu^2 - \kappa^2}{\nu^2}$ and $x = \sqrt{1 - \frac{z_{\rm min}^2}{z_{\rm max}^2}}$. The expansion of the conserved charges to $\mathcal{O}(z_{\rm min}^2)$ and $\mathcal{O}(z_{\rm max}^2)$, gives rise to

$$
E - J \approx 2T\left(\left|\sin\frac{p}{2}\right| - \frac{z_{\rm max} z_{\rm min}^2}{4}\right). \tag{4.4}
$$

The leading behaviors of E and $z_{\rm max}$ are given by

$$
\begin{aligned}
E &\approx 2T z_{\rm max} \log\frac{4 z_{\rm max}}{z_{\rm min}}, \\
z_{\rm max} &\approx \left|\sin\frac{p}{2}\right|.
\end{aligned} \tag{4.5}
$$

Using these relations, we finally obtain the approximate form of the dispersion relation for a magnon with the finite size correction

$$
\begin{aligned}
E - J &= 2T\left(\left|\sin\frac{p}{2}\right| - 4\left|\sin^3\frac{p}{2}\right| e^{-E/(T|\sin\frac{p}{2}|)}\right) \\
&= \sqrt{2\lambda}\left(\left|\sin\frac{p}{2}\right| - 4\left|\sin^3\frac{p}{2}\right| e^{-2E/(\sqrt{2\lambda}|\sin\frac{p}{2}|)}\right).
\end{aligned} \tag{4.6}
$$

For the infinite size case E and $J \to \infty$, this gives the same result obtained in the previous section. The second term in the above equation is the finite size correction. Note that this result is the finite correction for a giant magnon dual to open spin chain and that this correction corresponds to half of that for the closed string [21].

Spike case: in this section, we will calculate the finite size effect for a spike. Note that as previously mentioned, for considering a positive angular momentum, we should consider the angular momentum for a spike as $J' \equiv -J$ by changing the directions of rotation. Keeping this in mind, we now start to calculate the finite size effect for a spike. For the spike, $\sin\theta_{\rm min}$ and $\sin\theta_{\rm max}$ are given by

$$
\begin{aligned}
\sin\theta_{\rm min} &\equiv \sqrt{1 - z_{\rm max}^2} = \frac{\kappa}{\nu}, \\
\sin\theta_{\rm max} &\equiv \sqrt{1 - z_{\rm min}^2} = \frac{c}{\kappa b}.
\end{aligned} \tag{4.7}
$$

Note that in the infinite size limit, this parameterization reduces to the one used in the previous section, $\kappa = \frac{c}{b}$ and $\sin\theta_{\rm min} = \frac{c}{b\nu}$.

Using these, the conserved charges can be rewritten as

$$\begin{aligned} E &= 2T\,\frac{z_{\rm max}^2 - z_{\rm min}^2}{z_{\rm max}\sqrt{1-z_{\rm max}^2}}\,K(x), \\ J' &= 2T\,\frac{1}{z_{\rm max}}\left(z_{\rm max}^2 E(x) - z_{\rm min}^2 K(x)\right), \\ \frac{\Delta\phi}{2} &= \frac{\sqrt{1-z_{\rm min}^2}}{z_{\rm max}\sqrt{1-z_{\rm max}^2}}\left[K(x) - \Pi\left(\frac{z_{\rm max}^2 - z_{\rm min}^2}{\sqrt{z_{\rm max}^2-1}}; x\right)\right]. \end{aligned} \tag{4.8}$$

Here, the angular momentum J' is given by

$$J' \approx 2Tz_{\rm max} - \left(\frac{1}{2} + \log\frac{4z_{\rm max}}{z_{\rm min}}\right)\frac{Tz_{\rm min}^2}{z_{\rm max}}, \tag{4.9}$$

at $\mathcal{O}(z_{\rm min}^2)$. Note that for the infinite size limit, $z_{\rm min} \to 0$, the second term in the right hand side vanishes, so J' is always finite as it should be. The dispersion relation for a spike $E - \Delta\phi$ is given by up to $\mathcal{O}(z_{\rm min}^3)$ and $\mathcal{O}(z_{\rm max}^3)$ as

$$\begin{aligned} E - T\Delta\phi \approx\ & 2T\arcsin z_{\rm max} \\ & - \left[\left(\frac{1}{2z_{\rm max}} - \frac{z_{\rm max}}{4}\right)T + \left(\frac{1}{z_{\rm max}} + \frac{z_{\rm max}}{2}\right)T\log\frac{4z_{\rm max}}{z_{\rm min}}\right]z_{\rm min}^2. \end{aligned} \tag{4.10}$$

Using eq. (4.9), $2T\arcsin z_{\rm max}$ can be approximately rewritten as

$$2T\arcsin z_{\rm max} \approx 2T\arcsin\frac{J'}{2T} + \frac{\frac{1}{2}+\log\frac{4z_{\rm max}}{z_{\rm min}}}{z_{\rm max}\sqrt{1-\frac{J'^2}{4T^2}}}\,Tz_{\rm min}^2. \tag{4.11}$$

To rewrite the dispersion relation in terms of the physical quantities, $z_{\rm min}$ and $z_{\rm max}$ should be replaced with E and J'. From the leading term of E we obtain

$$z_{\rm min} = 4z_{\rm max}e^{-E/2Tz_{\rm max}} \tag{4.12}$$

and the leading term of J' gives

$$z_{\rm max} = \frac{J'}{2T}. \tag{4.13}$$

Using these, we finally obtain the dispersion relation for a finite size spike solution

$$\begin{aligned} E - T\Delta\phi \approx\ & 2T\arcsin\frac{J'}{2T} - \left[\left(4 - \frac{4}{\sqrt{1-(J'/2T)^2}} - \frac{J'^2}{2T^2}\right)J'\right. \\ & \left. + \left(8 - \frac{8}{\sqrt{1-(J'/2T)^2}} + \frac{J'^2}{T^2}\right)E\right]e^{-2E/J'}. \end{aligned} \tag{4.14}$$

In the infinite size limit, since the spike has infinite E and $\Delta\phi$ with the finite J', the above result gives rise to the same dispersion relation obtained in previous section

$$E - T\Delta\phi \approx 2T\arcsin z_{\rm max} = 2T\left(\frac{\pi}{2} - \theta_{\rm min}\right). \tag{4.15}$$

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The second term in eq. (4.14) corresponds to the finite size effect for a spike. For $z_{\max} \ll 1$ ($T \gg J'$), the above dispersion relation reduces to

$$E - T\Delta\phi \approx 2T \arcsin \frac{J'}{2T} + \left(1 + \frac{3J'E}{8T^2}\right) \frac{J'^3}{T^2}\, e^{-2E/J'}, \tag{4.16}$$

where $T = \frac{\sqrt{2\lambda}}{2}$.

5. Discussions

We have studied, in this paper, the rotating string in the diagonal SU(2) inside $\mathrm{AdS}_4 \times \mathbf{CP}^3$. We have solved the most general form of the equations of motion of the rotating string on $R \times S^2 \times S^2$, and have found out the most general form of all conserved charges. We have shown the existence of both the already known giant magnon, and the new spike solutions for the string and have found out the relevant dispersion relation among various charges in the infinite size limit. Furthermore, we have studied the finite size correction in both cases. It will be interesting to find out a three spin giant magnon with one spin along the AdS_4 and two angular momenta one in each of S^2 and study the dual gauge theory. Another interesting aspect will be to write down the semiclassical scattering of the giant magnon and spike solutions on $\mathrm{AdS}_4 \times \mathbf{CP}^3$. We wish to come back to some of these issues in future.

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